विध्न विचारत भीरु जन, नहीं आरम्भे काम, विपति देख छोड़े तुरंत मध्यम मन कर श्याम।
पुरुष सिंह संकल्प कर, सहते विपति अनेक, 'बना' न छोड़े ध्येय को, रघुबर राखे टेक।।

रचितः मानव धर्म प्रणेता

सद्गुरु श्री रणछोड़दासजी महाराज

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Q.1 A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\frac{dx}{dt} = c = \frac{dy}{dt}$. Where a, b and c are constants. The acceleration of the particle is

(A) $(6ac^2x + 2bc^2)\hat{k}$ (B) $(2ax^2 + 6by^2)\hat{k}$ (C) $(4bc^2x + 6ac^2)\hat{k}$ (D) $(bc^2x + 2by)\hat{k}$

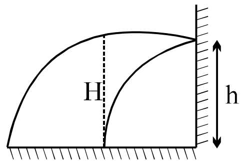
[Sol. $z = ax^3 + by^2$, $\frac{dz}{dt} = 3ax^2\frac{dx}{dt} + 2by\frac{dy}{dt}$

$$\frac{dz}{dt} = 3acx^2 + 2bcy, \quad \frac{d^2z}{dt^2} = 6acx\frac{dx}{dt} + 2bc\frac{dy}{dt}$$

$$\frac{d^2z}{dt^2} = (6ac^2x + 2bc^2)$$

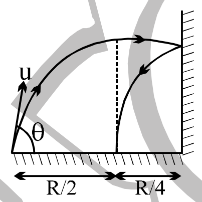
$$\vec{a} = (6ac^2x + 2bc^2)\hat{k}$$]

Q.2 A stone is projected from a horizontal plane. It attains maximum height 'H' & strikes a stationary smooth wall & falls on the ground vertically below the maximum height. Assume the collision to be elastic the height of the point on the wall where ball will strike is:



(A) H/2 (B) H/4
(C) 3H/4 (D) none of these

[Sol. Because horizontal velocity is constant so



$$T = \frac{2u\sin\theta}{g}$$

given $H = \frac{u^2\sin^2\theta}{2g}$, $u\sin\theta = \sqrt{2gH}$

$$T = \frac{2\sqrt{2gH}}{g}$$ at the time of hitting the wall

The horizontal distance covered is $\frac{3R}{4}$, so time taken to cover horizontal distance $\frac{3R}{4}$

$$T' = \frac{3T}{4} = 3\sqrt{\frac{H}{2g}}, \quad h = \sqrt{2gH} \times 3\sqrt{\frac{H}{2g}} - \frac{1}{2}\times g\times a\times\frac{H}{2g}$$

$$h = \frac{3H}{4}$$]

Q.3 A man in a balloon rising vertically with an acceleration of 4.9 m/s^2 releases a ball 2 seconds after the balloon is let go from the ground. The greatest height above the ground reached by the ball is ($g = 9.8$ m/s^2)

(A) 14.7 m (B) 19.6 m (C) 9.8 m (D) 24.5 m

[Sol. $v = 0 + 4.9 \times 2 = 9.8$ m/s

$$h_1 = \frac{1}{2} \times 4.9 \times 4 = 9.8 \text{ m}$$

$$0 = v^2 - 2gh_2$$

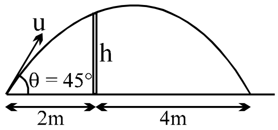
$$h_2 = \frac{v^2}{2g} = \frac{9.8\times9.8}{2\times9.8} = 4.9$$

$H = h_1 + h_2 = 9.8 + 4.9 = 14.7$ m]

Q.4 A particle is projected at an angle of 45° from a point lying 2 m from the foot of a wall. It just touches the top of the wall and falls on the ground 4 m from it. The height of the wall is

(A) 3/4 m (B) 2/3 m (C) 4/3 (D) 1/3 m

[Sol. $R = 6 = \frac{u^2 \sin 2\theta}{g}$, $\theta = 45°$



$60 = u^2$, $u = \sqrt{60}$

$u\cos\theta = \sqrt{30}$, $u\sin\theta = \sqrt{30}$

$t = \frac{2}{\sqrt{30}}$,

$h = \sqrt{30} \times \frac{2}{\sqrt{30}} - \frac{1}{2}\times 10 \times \frac{4}{30}$

$h = 2 - \frac{2}{3} = \frac{4}{3}$ m,

$h = \frac{4}{3}$ m]

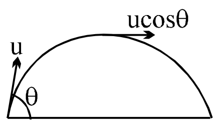
Q.5 The velocity at the maximum height of a projectile is half its initial velocity of projection. Its range on the horizontal plane is

(A) $\frac{\sqrt{3}u^2}{2g}$ (B) $\frac{u^2}{2g}$ (C) $\frac{3u^2}{2g}$ (D) $\frac{3u^2}{g}$

[Sol. Given:— $u\cos\theta = u/2$

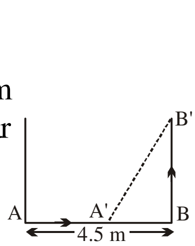
$\cos\theta = 1/2$, $\theta = 60°$

$$R = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2u^2\times\frac{\sqrt{3}}{2}\times\frac{1}{2}}{g}$$



$\Rightarrow$ $R = \frac{\sqrt{3}u^2}{2g}$]

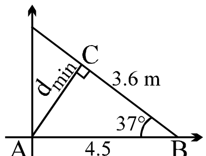
Q.6 Two particles instantaneously at A & B respectively 4.5 meters apart are moving with uniform velocities as shown in the figure. The former towards B at 1.5 m/sec and the latter perpendicular to AB at 1.125 m/sec. The instant when they are nearest is:



(A) 2 sec (B) 3 sec (C) 4 sec (D) $1\frac{23}{25}$ sec

[Sol. $\vec{V}_1 = 1.5\hat{i}$, $\vec{V}_2 = 1.125\hat{j}$

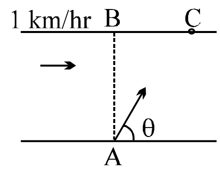
$\vec{V}_{21} = 1.125\hat{j} - 1.5\hat{i}$



$|\vec{V}_{21}| = \sqrt{(1.125)^2 + (1.5)^2} = 1.875$ m/s

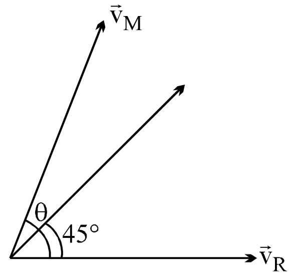
$t = \frac{3.6}{1.875} = 1.92$ sec $= 1\frac{23}{25}$]

Q.7 A river is flowing with a speed of 1 km/hr. A swimmer wants to go to point 'C' starting from 'A'. He swims with a speed of 5 km/hr, at an angle θ w.r.t. the river. If AB = BC = 400 m. Then the value of θ is:



(A) 37° (B) 30° (C) 53° (D) 45°

[Sol. Condition for reaching the point C



$\tan 45° = \frac{v_y}{v_x}$, $v_y = v_x$

$(v_R + v_M\cos\theta) = v_M\sin\theta$

$1 + 5\cos\theta = 5\sin\theta$

$1 + 5\cos\theta = 5\sqrt{1-\cos^2\theta}$

On squaring,

$1 + 25\cos^2\theta + 10\cos\theta = 25 - 25\cos^2\theta$

$50\cos^2\theta + 10\cos\theta - 24 = 0$

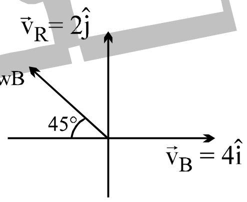
On solving,

$\Rightarrow \quad \theta = 53°$]

Q.8 A boat is moving towards east with velocity 4 m/s with respect to still water and river is flowing towards north with velocity 2 m/s and the wind is blowing towards north with velocity 6 m/s. The direction of the flag blown over by the wind hoisted on the boat is:

(A) north-west (B) south-east (C) $\tan^{-1}(1/2)$ with east (D) north

[Sol. $\vec{v}_{Bg} = \vec{v}_{BR} + \vec{v}_{Rg} = 4\hat{i} + 2\hat{j}$

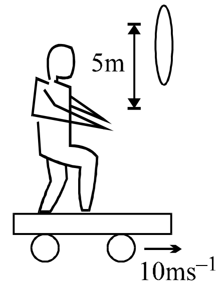


$\vec{v}_{wg} = 6\hat{i}$

$\vec{v}_{wB} = \vec{v}_w - \vec{v}_B = 6\hat{i} - 4\hat{i} - 2\hat{j} = 2\hat{i} - 2\hat{j}$

Direction will be north-west]

Q.9 A girl is riding on a flat car travelling with a constant velocity 10 ms^{-1} as shown in the fig. She wishes to throw a ball through a stationary hoop in such a manner that the ball will move horizontally as it passes through the hoop. She throws the ball with an initial speed $\sqrt{136}$ ms^{-1} with respect to car. The horizontal distance in front of the hoop at which ball has to be thrown is

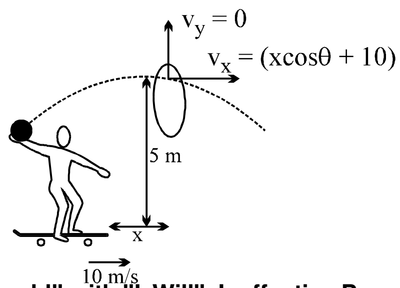


(A) 1m (B) 2m (C) 4m (D) 16m

[Sol. $x = (10 + \sqrt{136}\cos\theta)t$ ...(1)

$v_y^2 = 0 = 136\sin^2\theta - 2 \times 10 \times 5$ ...(2)

$\sin\theta = \frac{5}{\sqrt{34}}$, $\cos\theta = \frac{3}{\sqrt{34}}$



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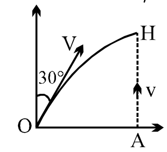
$5 = \sqrt{136} \times \frac{5}{\sqrt{34}} t - 5t^2$ ...(3)

$t^2 - 2t + 1 = 0 \quad \Rightarrow \quad t = 1$

So, x = 16 m]

Q.10 A particle is projected with a speed V from a point O making an angle of 30° with the vertical. At the same instant, a second particle is thrown vertically upward from a point A with speed v. The two particle reach H, the highest point on the parabolic path of the first particle simultaneously, then the ratio V/v

(A) $3\sqrt{2}$ (B) $2\sqrt{3}$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{2}$



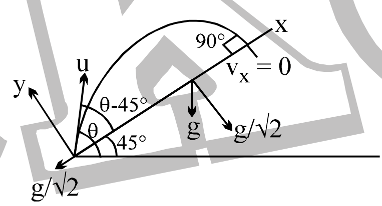
[Sol. $H = \frac{V^2 \sin^2 60°}{2g}$, $\sqrt{\frac{2gH \times 4}{3}} = V = \sqrt{\frac{8gH}{3}}$

$v = \sqrt{2gH}$, $\frac{V}{v} = \frac{\sqrt{8gH/3}}{\sqrt{2gH}} = \frac{2}{\sqrt{3}}$

$\frac{V}{v} = \frac{2}{\sqrt{3}}$]

Q.11 A particle is projected with a certain velocity at an angle θ above the horizontal from the foot of a given plane inclined at an angle of 45° to the horizontal. If the particle strike the plane normally then θ equals
(A) $\tan^{-1}(1/3)$ (B) $\tan^{-1}(1/2)$ (C) $\tan^{-1}(1/\sqrt{2})$ (D) $\tan^{-1} 3$

[Sol. $v_x = u_x - \frac{g}{\sqrt{2}} t$



$0 = u\cos(\theta - 45°) - \frac{g}{\sqrt{2}} t$

$t = \frac{\sqrt{2} u \cos(\theta - 45°)}{g}$

$y = u_y t - \frac{1}{2} \frac{g}{\sqrt{2}} t^2$, $y = 0$

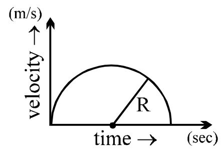
$\frac{2\sqrt{2} u_y}{g} = t = \frac{2\sqrt{2} u \sin(\theta - 45°)}{g}$

$\frac{\sqrt{2} u \cos(\theta - 45°)}{g} = \frac{2\sqrt{2} u \sin(\theta - 45°)}{g}$

$\frac{1}{2} = \tan(\theta - 45°)$

$\frac{\tan\theta - 1}{1 + \tan\theta} = \frac{1}{2} \quad \Rightarrow \quad \tan\theta = 3 \quad \Rightarrow \quad \theta = \tan^{-1}(3)$]

Q.12 Velocity time graph of a particle is in shape of a semicircle of radius R as shown in figure. Its average acceleration from T = 0 to T = R is:



(A) 0 m/s^2 (B) 1 m/s^2

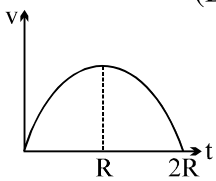
(C) R m/s^2 (D) 2R m/sec^2

[Sol. u = 0, V = R (at T = R)

T = R, V = u + at

R = 0 + a × R

a = 1 m/s^2



]

Q.13 A car is moving with uniform acceleration along a straight line between two stops X and Y. Its speed at X and Y are 2m/s and 14m/s. Then

(A) Its speed at mid point of XY is 15m/s

(B) Its speed at a point A such that XA : AY = 1 : 3 is 5m/s

(C) The time to go form X to the mixed point of XY is double of that to go from mid point to Y.

(D) The distance travel in first half of the total time is half of the distance travelled in the second half of the time.

[Sol. (A) Not possible if acceleration is const.

(B) Velocity at mid-point $v^2 = u + 2 \times a \times \frac{\ell}{4}$

$$106 = v^2 + 2 \times a \times \frac{3\ell}{4}$$

$$a = \frac{96}{\ell}, \; v = \sqrt{52}$$

(C) Possible

(D) Not possible]

Q.14 A particle having a velocity $v = v_0$ at t = 0 is decelerated at the rate $|a| = \alpha\sqrt{v}$, where α is a positive constant.

(A) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$

(B) The particle will come to rest at infinity.

(C) The distance travelled by the particle is $\frac{2v_0^{3/2}}{\alpha}$

(D) The distance travelled by the particle is $\frac{2}{3}\frac{v_0^{3/2}}{\alpha}$

[Sol. (A) $a = -\alpha\sqrt{v}$

$$\frac{dv}{dt} = -\alpha v^{1/2}$$

$$\int_{v_0}^{0} \frac{dv}{v^{1/2}} = \int_0^t -\alpha dt$$

$$\left[2v^{1/2}\right]_{v_0}^{0} = -\alpha t$$

$$2[-v_0^{1/2}] = -\alpha t$$

$$t = \frac{2\sqrt{v_0}}{\alpha}$$

(D) Velocity at any time t is

$$\int_{v_0}^{v} \frac{dv}{v^{1/2}} = -\int_0^t \alpha\, dt$$

$$[2v^{1/2}]_{v_0}^{v} = -\alpha t$$

$$2[v^{1/2} - v_0^{1/2}] = -\alpha t$$

$$v = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$$

$$v = \frac{d\alpha}{dt} = \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2$$

$$\int_0^{\alpha} d\alpha = \int_0^{\alpha} \left(\sqrt{v_0} - \frac{\alpha t}{2}\right)^2 dt$$

$$x = v_0 t + \frac{\alpha t^3}{12} - \frac{\alpha t^2 \sqrt{v_0}}{2}$$

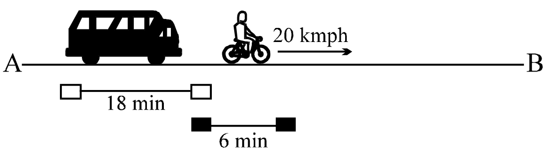
$$\text{at } t = \frac{2\sqrt{v_0}}{\alpha}$$

$$x = \frac{2}{3}\frac{v_0^{3/2}}{\alpha}$$]

Q.15 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with speed of 20km/h in the direction A to B, notices that a bus goes past him every $t_1 = 18$ minutes in the direction of motion, and every $t_2 = 6$ minutes in the opposite direction. What is the period T of the bus service? Assume that velocity of cyclist is less than velocity of bus

(A) 4.5 minutes (B) 24 minutes (C) 9 minutes (D) 12 minutes

[Sol.



$$(v - 20)\,\frac{18}{60} = d = vt$$

$$(v + 20)\,\frac{6}{60} = d = vt$$

$3v - 60 = v + 20$

$v = 40$ kmph

$(40 + 20) \times \frac{6}{60} = 40 \times T$

$6 = 40\ T \qquad \Rightarrow \qquad T = 6/40\ hr = 9\ min$]

Q.16 A body starts from rest with uniform acceleration. Its velocity after 2n second is v_0. The displacement of the body in last n seconds is

(A) $\frac{v_0(2n-3)}{6}$ (B) $\frac{v_0}{4n}(2n-1)$ (C) $\frac{3v_0 n}{4}$ (D) $\frac{3v_0 n}{2}$

[Sol. $v_0 = 0 + a \times 2n$ ...(1)

$S_{2n} = \frac{1}{2} a \times (2n)^2$

$S_n = \frac{1}{2} a \times (n)^2$

$S_{2n} - S_n = \frac{3}{2} an^2$ ...(2)

$= \frac{3}{2} \times \frac{v_0}{2n} \times n^2 = \frac{3v_0 n}{4}$]

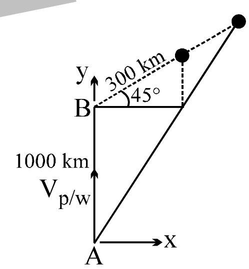
Q.17 An airplane pilot wants to fly from city A to city B which is 1000 km due north of city A. The speed of the plane in still air is 500 km/hr. The pilot neglects the effect of the wind and directs his plane due north and 2 hours later find himself 300km due north-east of city B. The wind velocity is

(A) 150km/hr at 45°N of E (B) 106km/hr at 45°N of E
(C) 150 km/he at 45°N of W (D) 106 km/hr at 45°N of W

[Sol. $V_{p/w} = 500\ kmph\ \hat{j}$

$V_{w/g} = v_x \hat{i} + v_y \hat{j}$

$V_{p/g} = v_x \hat{i} + (v_y + 500)\hat{j}$ kmph



In two hours

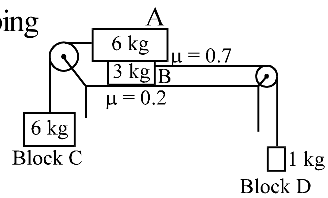
$S_{p/g} = 150\sqrt{2}\,\hat{i} + 1150\sqrt{2}\,\hat{j}$ km $= 2v_x\,\hat{i} + (2v_y + 1000)\,\hat{j}$

$\Rightarrow \qquad v_x = 75\sqrt{2}$

$\qquad v_y = 75\sqrt{2}$

$V_{w/g} = 75\sqrt{2}\,\hat{i} + 75\sqrt{2}\,\hat{j} = 150$ kmph at 45° N of E]

Q.18 An arrangement of the masses and pulleys is shown in the figure. Strings connecting masses A and B with pulleys are horizontal and all pulleys and strings are light. Friction coefficient between the surface and the block B is 0.2 and between blocks A and B is 0.7. The system is released from rest (use $g = 10\ m/s^2$).



(A) The magnitude of acceleration of the system is $2\ m/s^2$ and there is no slipping between block A and block B
(B) The magnitude of friction force between block A and block B is 42 N
(C) Acceleration of block C is $1\ m/s^2$ downwards
(D) Tension in the string connecting block B and block D is 12 N

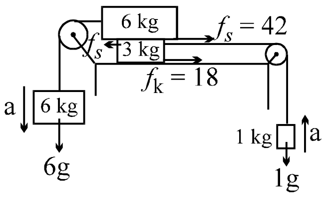
[Sol. (A) $6g - 18 - 1g = 16a$

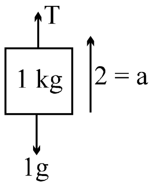
$a = 2\ m/s^2$ C moving down

(B) $f_s - 18 - 10 = 4 \times 2$

$f_s = 36$

(C) $a_c = 2\ m/s^2$ downward



(D) 

$T - 10 = 1 \times 2 \Rightarrow T = 12\ N$]

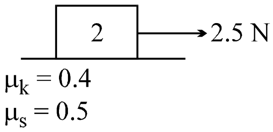
Q.19 A body of mass 2 kg is placed on a horizontal surface having kinetic friction 0.4 and static friction 0.5. If the force applied on the body is 2.5 N, the frictional force acting on the body will be ($g = 10\ m/s^2$)

(A) 8 N (B) 10 N (C) 20 N (D) 2.5 N

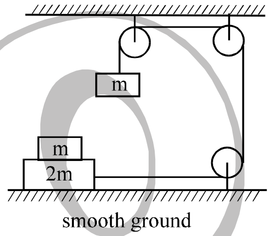
[Sol. $f_{l \cdot s} = \mu_s mg = 0.5 \times 2g = 10\ N$

Block is stationary $P < f_{l \cdot s}$

$\Rightarrow$ friction force $f = P = 2.5\ N$]



Q.20 In the arrangement shown in figure, there is friction between the blocks of masses m and 2m which are in contact. The ground is smooth. The mass of the suspended block is m. The block of mass m which is kept on mass 2m is stationary with respect to block of mass 2 m. The force of friction between m and 2m is (pulleys and strings are light and frictionless) :

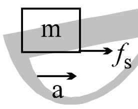


(A) $\frac{mg}{2}$ (B) $\frac{mg}{\sqrt{2}}$ (C) $\frac{mg}{4}$ (D) $\frac{mg}{3}$

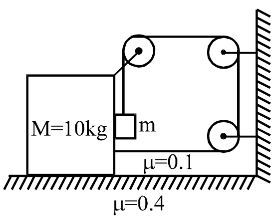
[Sol. $mg = 4ma$

$a = g/4$

$f_s = ma = mg/4$]



Q.21 The maximum value of m(in kg) so that the arrangement shown in the figure is in equilibrium is given by



(A) 2 (B) 2.5 (C) 3 (D) 3.5

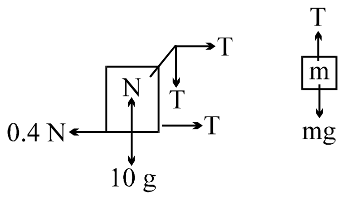
[Sol. Bigger block is not moving

$T = mg$ ...(1)

$2T = 0.4\ N$ ...(2)

$T + 10\ g = N$ ...(3)

String



$$\frac{2T}{T+100} = 0.4$$

$1.6\ T = 40$

$T = 25$

$\Rightarrow$ $m = 2.5\ kg$]

Q.22 Two blocks, A and B, of same masses, are resting in equilibrium on an inclined plane having inclination with horizontal = α (>0). The blocks are touching each other with block B higher than A. Coefficient of static friction of A with incline = 1.2 and of B = 0.8. If motion is not imminent,

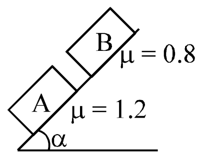
(A) $\alpha < 30°$ (B) $(\text{Friction})_A > (\text{Friction})_B$

(C) $\alpha < 45°$ (D) $(\text{Friction})_A = (\text{Friction})_B$

[Sol.

$2mg\sin\alpha < \underset{(f_A + f_B)_{max}}{2mg\cos\alpha}$

$\tan\alpha < 1 \quad \Rightarrow \quad \alpha < 45°$



$f_A = mg\sin\alpha + N$

$f_B = mg\sin\alpha - N$

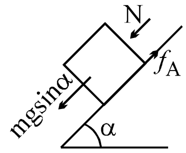
To show $N \neq 0$

For A

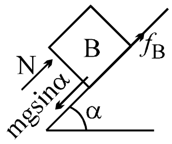
$f_{b_{max}} = 0.8\ mg\cos\alpha$

$\Rightarrow \quad N = mg[\sin\alpha - 0.8\cos\alpha]$

$N = mg\sec\alpha[\tan\alpha - 0.8]$



FBD of A

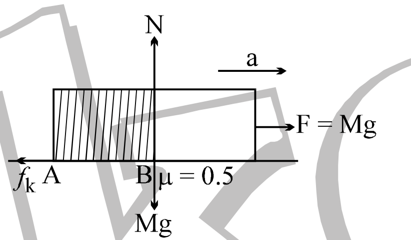


For $\alpha > \tan^{-1}(0.8)$

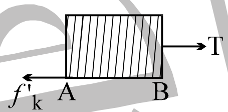
$f_A > f_B$ for $\alpha \le \tan^{-1}(0.8)\ f_A = f_B$]

Q.23 A rope of length L and mass M is being puled on a rough horizontal floor by a constant horizontal force F = Mg. The force is acting at one end of the rope in the same direction as the length of the rope. The coefficient of kinetic friction between rope and floor is 1/2. Then, the tension at the midpoint of the rope is

(A) Mg / 4 (B) 2Mg / 5 (C) Mg / 8 (D) Mg / 2



[Sol. $a = \dfrac{F - \mu N}{M} = \dfrac{Mg - 0.5Mg}{M} = g/2$



$T - \mu Mg/2 = Ma/2$

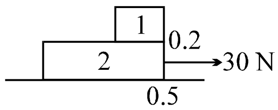
$T - Mg/4 = Mg/4$

$T = Mg/2$]

Q.24 A plank of mass 2kg and length 1 m is placed on a horizontal floor. A small block of mass 1 kg is placed on top of the plank, at its right extreme end. The coefficient of friction between plank and floor is 0.5 and that between plank and block is 0.2. If a horizontal force = 30 N starts acting on the plank to the right, the time after which the block will fall off the plank is ($g = 10\ m/s^2$)

(A) (2/3) s (B) 1.5 s (C) 0.75 s (D) (4/3) s

[Sol. $a_{1/g} = 2\ m/s^2$



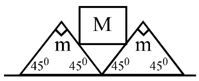
$a_{2/g} = \dfrac{30 - 2 - 15}{2} = \dfrac{13}{2} = 6.5\ m/s^2$

$a_{2/1} = 4.5$

$s_{2/1} = \dfrac{1}{2} a_{2/1} t^2 \qquad 1 = \dfrac{1}{2} \times 4.5 \times t^2$

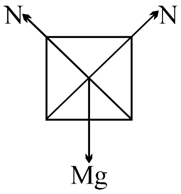
$t = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$ sec]

Q.25 Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown. Assume no friction between the cube and the wedges, but a coefficient of static friction $\mu < 1$ between the wedges and the floor. What is the largest M that can be balanced as shown without motion of the wedges?



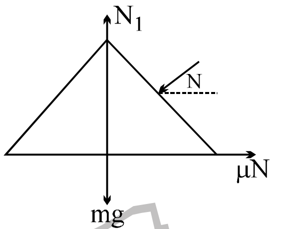
(A) $\frac{m}{\sqrt{2}}$ (B) $\frac{\mu m}{\sqrt{2}}$ (C) $\frac{2\mu m}{1-\mu}$ (D) All M will balance

[Sol. $2N\cos 45 = Mg$ ...(1)



$\frac{N}{\sqrt{2}} + mg = N_1$ ...(2)

$\frac{N}{\sqrt{2}} = \mu N_1$ ...(3)



$$\frac{N}{\sqrt{2}} = \mu\left[\frac{N}{\sqrt{2}} + mg\right]$$

$$\Rightarrow N = \left(\frac{\sqrt{2}\mu}{1-\mu}\right)mg$$

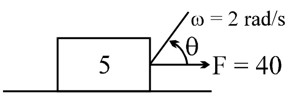
$$\therefore \sqrt{2}\,N = \frac{2\mu}{1-\mu}\,mg = Mg$$

$$M = \frac{2\mu m}{1-\mu}$$]

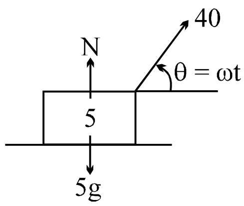
Q.26 A force F acting on a particle of mass 5 kg placed on a smooth horizontal surface. F = 40 N remains constant but its vector rotates in a vertical plane at an angular speed 2 rad/sec. If a t = 0, vector F is horizontal, find the velocity of block at $t = \pi/4\omega$ sec.

(A) 1 m/s (B) $\sqrt{2}$ m/s (C) 2 m/s (D) $2\sqrt{2}$ m/s

[Sol.



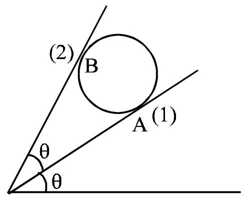
At any time t



$$a = \frac{40\cos\omega t}{5} = 8\cos 2t$$

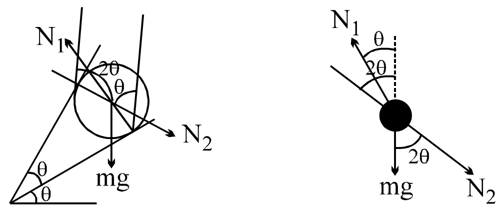
$$dv = 8\int_0^{\pi/8}\cos 2t\,dt = \frac{8}{2}\left[\sin 2t\right]_0^{\pi/8} = 4\left[\sin\left(\frac{\pi}{4}\right) - 0\right] = 2\sqrt{2}\ \text{m/s}$$]

Q.27 A sphere of mass m is kept between two inclined walls, as shown in the figure. If the coefficient of friction between each wall and the sphere is zero, then the ratio of normal reaction (N_1/N_2) offered by the walls 1 and 2 on the sphere will be



(A) $\tan\theta$
(B) $\tan 2\theta$
(C) $2\cos\theta$
(D) $\cos 2\theta$

[Sol.



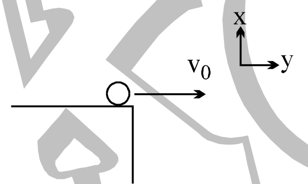
$N_1\cos\theta = mg + N_2\cos 2\theta$ ...(1)
$N_1\sin\theta = N_2\sin 2\theta$ ...(2)

By eq (2)

$$\frac{N_1}{N_2} = 2\cos\theta \quad]$$

Q.28 A particle is projected horizontally from the top of a tower with a velocity v_0. If v be its velocity at any instant, then the radius of curvature of the path of the particle at the point (where the particle is at that instant) is directly proportional to:
(A) v^3
(B) v^2
(C) v
(D) 1/v

[Sol. $\vec{v} = v_0\hat{i} - gt\hat{j}$ and $\vec{a} = -g\hat{j}$



$\therefore$ Component of $\vec{a} \perp$ to $\vec{v} = \vec{a} - \left(\frac{\vec{a}\cdot\vec{v}}{v^2}\right)\vec{v}$

i.e. $\vec{a}_\perp = \left(-\frac{v_0 g}{v_0^2 + g^2t^2}\right)(gt\hat{i} + v_0\hat{j})$

$\therefore$ $|\vec{a}_\perp| = \frac{v_0 g}{\sqrt{v_0^2 + gt^2}}$

Also, $r = \frac{|\vec{v}|^2}{|a_\perp|} = \frac{(v_0^2 + g^2t^2)^{3/2}}{v_0 g} = \frac{v^3}{v_0 g}$

$\therefore$ $r \propto v^3$

$\therefore$ Option (A) is correct]

Q.29 There are two massless springs A and B of spring constant K_A and K_B respectively and $K_A > K_B$. If W_A and W_B be denoted as work done on A and work done on B respectively, then
(A) If they are compressed to same distance, $W_A > W_B$
(B) If they are compressed by same force (upto equilibrium state) $W_A < W_B$
(C) If they are compressed by same distance, $W_A = W_B$
(D) If they are compressed by same force (upto equilibrium state) $W_A > W_B$

[Sol. For same compression x_0 (say)

$$W_A = \frac{1}{2}k_A x_0^2 \;\; \& \;\; W_B = \frac{1}{2}k_B x_0^2$$

$\Rightarrow \quad W_A > W_B \quad [\therefore k_A > k_B]$

for same force at equilibrium force = F_0

$$x_A = \frac{F_0}{k_A},\ x_B = \frac{F_0}{k_B}$$

$$\therefore \quad W_A = \frac{1}{2}k_A x_A^2 = \frac{F_0^2}{2k_A}$$

Similarly, $W_B = \dfrac{F_0^2}{2k_B}$

$\Rightarrow \quad W_B > W_A$

$\therefore$ (A) & (B) are correct options]

Q.30 A horizontal curve on a racing track is banked at a 45° angle. When a vehicle goes around this curve at the curve's safe speed (no friction needed to stay on the track), what is its centripetal acceleration?
(A) g (B) 2g (C) 0.5g (D) none

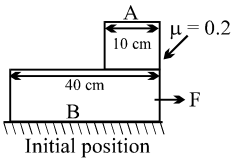
Q.31 Power delivered to a body varies as $P = 3t^2$. Find out the change in kinetic energy of the body from t = 2 to t = 4 sec.
(A) 12 J (B) 56 J (C) 24 J (D) 36 J

[Sol. Here power delivered is
$P = 3t^2$
If this power results into only kinetic energy change then

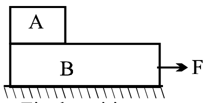
$$\Delta KE = \int_2^4 P\,dt = \int_2^4 3t^2\,dt = 3\left|\frac{t^3}{3}\right|_2^4 = (4^3 - 2^3)\ J = 56\ J$$

Power delivered will cause this maximum change in K.E.
(B) is correct option]

Q.32 A block 'A' of mass 45 kg is placed on a block 'B' of mass 123 kg. Now block 'B' is displaced by external agent by 50 cm horizontally towards right. During the same time block 'A' just reaches to the left end of block B. Initial & final position are shown in figure. Refer to the figure & find the workdone by frictional force on block A in ground frame during above time.



Initial position



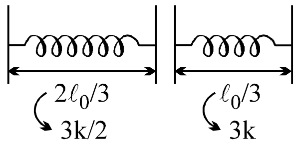
Final position

(A) – 18 Nm (B) 18 Nm (C) 36 Nm (D) – 36 Nm

[Sol. Here blocks are moving w.r.t. each other, hence friction force = 0.2 × 45 × 10 = 90 N
Given block 'B" moves 50 cm
Also given that block A moves (40 – 10) cm back w.r.t. block 'B'
$\therefore$ Forward movement of block A in ground frame = 50 – 30 cm = 20 cm
$\therefore$ Work done by friction force = 90 × 0.2 J = 18 J
Work done is positive
$\therefore$ Option (B) is correct]

Q.33 A spring of force constant k is cut in two part at its one third length. when both the parts are stretched by same amount. The work done in the two parts, will be
(A) equal in both (B) greater for the longer part
(C) greater for the shorter part (D) data insufficient.

[Sol. When a spring is cut into two parts each part has spring constant more than that of original spring. If k = spring constant & ℓ_0 = natural length, then for cut parts



If they are stretched by same amount then work done in shorter part will be double than that in the case of longer part.

$\therefore$ Option (C) is correct]

Q.34 The horsepower of a pump of efficiency 80%, which sucks up water from 10 m below ground and ejects it through a pipe opening at ground level of area 2 cm^2 with a velocity of 10 m/s, is about

(A) 1.0 hp (B) 0.5 hp (C) 0.75 hp (D) 4.5 hp

[Sol. Here,

area = 2 cm^2 = 2×10^{-4} m^2

velocity = 10 m/s

$\therefore$ Volume flow rate = 2×10^{-3} m^{3}s^{-1} = vρgh

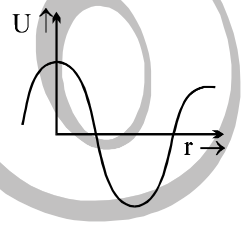
$\therefore$ Energy required per second = $100 \times 10^3 \times 2 \times 10^{-3}$ J = 2×100 J = 200 J

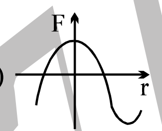
$\because$ Efficiency is 80%

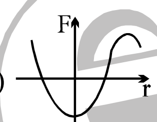
$\therefore$ Power of pump = 250 W

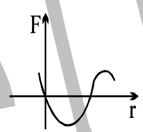
Hence (B) is correct option]

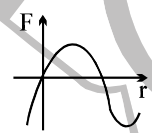
Q.35 Potential energy and position for a conservative force are plotted in graph shown. Then force position graph can be



(A)


(B)


(C)


(D)


[Sol. Here by the graph we can say that

$U = U_0 \cos r$

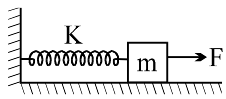
$\therefore$ $F = -\dfrac{dU}{dr} = -U_0(-\sin r)$

$F = U_0 \sin r$

Hence correct option is (D)]

Q.36 A constant force produces maximum velocity V on the block connected to the spring of force constant K as shown in the fig. When the force constant of spring becomes 4K, the maximum velocity of the block is

(A) $\dfrac{V}{4}$ (B) 2V (C) $\dfrac{V}{2}$ (D) V



[Sol. Block will gain maximum velocity at the point of equilibrium

In first case equilibrium elongation = $\dfrac{F}{k}$

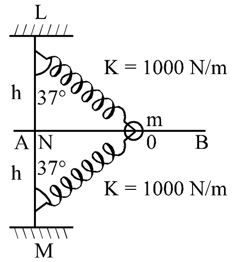
$\therefore$ $F \cdot \dfrac{F}{k} - \dfrac{1}{2}k\left(\dfrac{F}{k}\right)^2 = \dfrac{1}{2}mV^2 \quad \Rightarrow \quad V = \sqrt{\dfrac{F^2}{mk}}$

In second case equilibrium elongation = $\dfrac{F}{4k}$

$$F\cdot\frac{F}{4k}-\frac{1}{2}\times 4k\left(\frac{F}{4k}\right)^2=\frac{1}{2}mV'^{\,2}\quad\Rightarrow\quad V'=\sqrt{\frac{F^2}{4mk}}=\frac{V}{2}$$

$\therefore$ (C) is correct option]

Q.37 A bead of mass 5kg is free to slide on the horizontal rod AB. They are connected to two identical springs of natural length h ms. as shown. If initially bead was at O & M is vertically below L then, velocity of bead at point N will be



(A) 5h m/s (B) 40h/3 m/s
(C) 8h m/s (D) none of these

[Sol. Natural length of each spring is h

$\therefore$ elongation in each spring = $\dfrac{h}{\cos 37^\circ}-h=\dfrac{h}{4}$

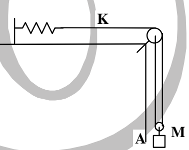
& applying work-energy theorem

$$\frac{1}{2}mv^2=2\times\frac{1}{2}k\left(\frac{h}{4}\right)^2$$

v = 5h m/s

$\therefore$ Option (A) is correct]

Q.38 Block A in the figure is released from rest when the extension in the spring is x_0. The maximum downwards displacement of the block is



(A) $\dfrac{Mg}{2K}-x_0$ (B) $\dfrac{Mg}{2K}+x_0$ (C) $\dfrac{2Mg}{K}-x_0$ (D) $\dfrac{2Mg}{K}+x_0$

[Sol. Let the block move 'x' downward then elongation in spring is '2x'

$$\therefore\quad \frac{1}{2}k(x_0+2x)^2-\frac{1}{2}k\,x_0^2=Mgx$$

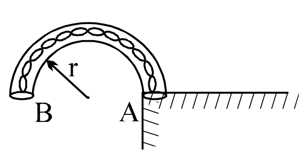
$$\Rightarrow k\,x_0^2+4kxx_0+4kx^2-k\,x_0^2=2Mgx$$

$$\because\quad x\neq 0\quad\Rightarrow\quad x_0+x=\frac{Mg}{2k}$$

$$\therefore\quad x=\frac{Mg}{2k}-x_0$$

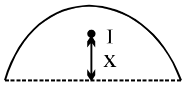
$\therefore$ Option (A) is correct]

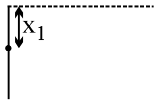
Q.39 A smooth semicircular tube AB of radius r is fixed in a vertical plane and contains a heavy flexible chain of length π r and weight W= π r as shown. Assuming a slight disturbance to start the chain in motion, the velocity v with which it will emerge from the open end B of the tube is



(A) $\dfrac{4gr}{\pi}$ (B) $\dfrac{2gr}{\pi}$ (C) $\sqrt{2gr\left(\dfrac{2}{\pi}+\pi\right)}$ (D) $\sqrt{2gr\left(\dfrac{2}{\pi}+\dfrac{\pi}{2}\right)}$

[Sol. Initial CM position Final CM position





$x = \dfrac{2r}{\pi}$ $\qquad\qquad x_1 = \dfrac{\pi r}{2}$

$\therefore$ Δh for CM $= x + x_1$

$\Delta PE = \Delta KE \quad \Rightarrow \quad W\Delta h = \dfrac{1}{2}\dfrac{W}{g}U^2$

$$U^2 = 2gr\left(\frac{2}{\pi} + \frac{\pi}{2}\right)$$

$$U = \sqrt{2gr\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$$

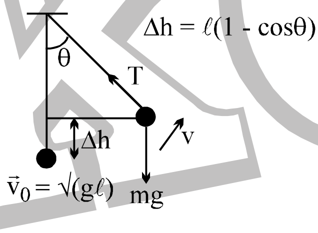
$\therefore$ Option (D) is correct]

Q.40 A heavy particle hanging from a string of length l is projected horizontally with speed $\sqrt{gl}$. The speed of the particle at the point where the tension in the string equals weight of the particle is:

(A) $\sqrt{2gl}$ (B) $\sqrt{3gl}$ (C) $\sqrt{gl/2}$ (D) $\sqrt{gl/3}$

[Sol. Speed at bottom $= \sqrt{g\ell} < \sqrt{2g\ell}$

$mg\ell(1 - \cos\theta) = \dfrac{1}{2}mg\ell - \dfrac{1}{2}mv^2$ ...(1)



Also, $T - mg\cos\theta = \dfrac{mv^2}{\ell}$

But $T = mg$

$\therefore$ $\dfrac{mv^2}{\ell} = mg - mg\cos\theta$

i.e. $\dfrac{1}{2}mv^2 = \dfrac{mg\ell}{2}(1 - \cos\theta)$

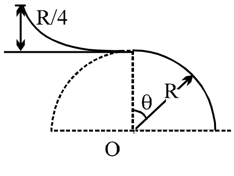
$\therefore$ eq^n (1) $\Rightarrow$ $mg\ell(1 - \cos\theta) = \dfrac{1}{2}mg\ell - \dfrac{1}{2}mg\ell(1 - \cos\theta)$

$$1 - \cos\theta = \frac{1}{3} \quad \Rightarrow \quad \cos\theta = \frac{2}{3}$$

$\therefore$ $v = \sqrt{g\ell/3}$

$\therefore$ Option (D) is correct]

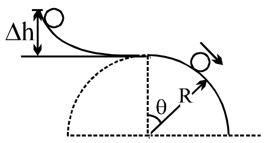
Q.41 A skier plans to ski a smooth fixed hemisphere of radius R. He starts from rest from a curved smooth surface of height (R/4). The angle θ at which he leaves the hemisphere is



(A) $\cos^{-1}(2/3)$ (B) $\cos^{-1}(5/\sqrt{3})$

(C) $\cos^{-1}(5/6)$ (D) $\cos^{-1}(5/2\sqrt{3})$

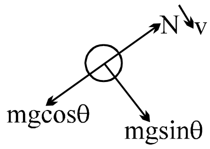
[Sol. $\Delta h = \frac{R}{4} + R(1 - \cos\theta)$



$\frac{1}{2}mv^2 = mg\Delta h = \frac{mgR}{4}\{1 + 4(1 - \cos\theta)\}$

$\therefore \quad \frac{mv^2}{R} = \frac{mg}{2}(5 - 4\cos\theta)$

$mg\cos\theta - N = \frac{mv^2}{R}$



$mg\cos\theta = \frac{mg}{2}(5 - 4\cos\theta)$

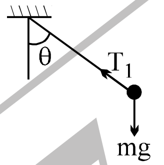
$\cos\theta = 5/6$

$\therefore$ Option (C) is correct]

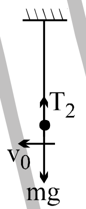
Q.42 A simple pendulum swings with angular amplitude θ. The tension in the string when it is vertical is twice the tension in its extreme position. Then, cos θ is equal to

(A) 1 / 3 (B) 1 / 2 (C) 2 / 3 (D) 3 / 4

[Sol. At extreme v = 0 At vertical position



$T_1 = mg\cos\theta$



$\frac{1}{2}mv_0^2 = mg\ell(1-\cos\theta)$

$\& \ T_2 - mg = mv_0^2/\ell$

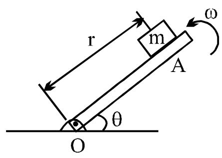
$\therefore \ T_2 = mg + 2mg(1-\cos\theta)$

Given $T_2 = 2T_1$ i.e. $mg(3 - 2\cos\theta) = 2mg\cos\theta$

$\therefore \quad 3 - 2\cos\theta = 2\cos\theta \quad \Rightarrow \quad \cos\theta = 3/4$

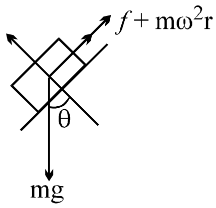
$\therefore$ (D) is correct option]

Q.43 The inclined plane OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity ω = 3 rad/sec. As it passes the position θ = 0, a small mass m = 1 kg is placed upon it at a radial distance r = 0.5 m. If the mass is observed to be at rest with respect to inclined plane. The value of static friction force at θ = 37° between the mass & the incline plane.



(A) 1.5 N (B) 3.5 N (C) 2.4 N (D) none

[Sol. Drawing the FBD in rotating frame we get



As the block is at rest, hence

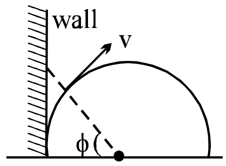
$mg\sin\theta = f + m\omega^2 r$

$\therefore \quad f = mg\sin\theta - m\omega^2 r = 1 \times 10 \times (3/5) - 1 \times 9 \times 0.5 = 6 - 4.5 = 1.5$

Therefore, force of friction (static) = 1.5 N

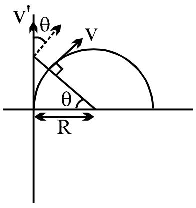
$\therefore$ (A) option is correct]

Q.44 On a particle moving on a circular path with a constant speed v, light is thrown from a projectors placed at the centre of the circular path. The shadow of the particle is formed on the wall. the velocity of shadow up the wall is



(A) v $\sec^2\phi$ (B) v $\cos^2\phi$ (C) (A) v $\cos\phi$ (D) none

[Sol. **Method I**



$$\frac{v'\cos\theta}{R\sec\theta} = \frac{v}{R}$$

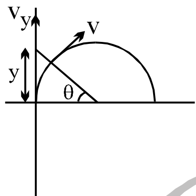
$v' = v\sec^2\theta$

Method II

$y = R\tan\theta$

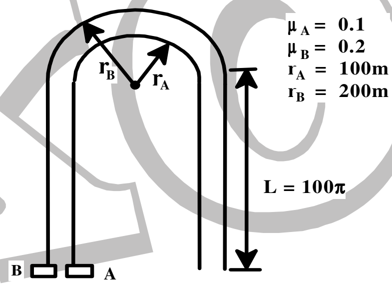
$$\frac{dy}{dt} = R\sec^2\theta = \frac{dv}{dt}$$

$V_y = R\sec^2\theta\,(\omega)$



$$V_y = R\sec^2\theta\left(\frac{v}{R}\right) = v\sec^2\theta]$$

Q.45 Two cars A and B start racing at the same time on a flat race track which consists of two straight sections each of length 100π and one circular section as in Fig. The rule of the race is that each car must travel at constant speed at all times without ever skidding



(A) car A completes its journey before car B
(B) both cars complete their journey in same time
(C) velocity of car A is greater than that of car B
(D) car B completes its journey before car A.

[Sol. $v \le \sqrt{\mu rg}$

$v_A \le \sqrt{0.1\times100\times10} = 10$ m/s

$v_B \le \sqrt{0.2\times200\times10} = 20$ m/s

$$t_A = \frac{200\pi + \pi(100)\,m}{10\,m/s} = 30\,\pi \text{ sec.}$$

$$t_B = \frac{200\pi + \pi(200)}{20\,m/s} = 20\,\pi \text{ sec.}]$$

